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# Extremal Black Holes As Fundamental Strings

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**Abstract**

We show that polarization dependent string-string scattering provides new evidence for the identification of the Dabholkar-Harvey (DH) string solution with the heterotic string itself. First, we construct excited versions of the DH solution which carry arbitrary left-moving waves yet are annihilated by half the supersymmetries. These solutions correspond in a natural way to Bogomolny-bound-saturating excitations of the ground state of the heterotic string. When the excited string solutions are compactified to four dimensions, they reduce to Sen's family of extremal black hole solutions of the toroidally compactified heterotic string. We then study the large impact parameter scattering of two such string solutions. We develop methods which go beyond the metric on moduli space approximation and allow us to read off the subleading polarization dependent scattering amplitudes. We find perfect agreement with heterotic string tree amplitude predictions for the scattering of corresponding string states. Taken together, these results clearly identify the string states responsible for Sen's extremal black hole entropy. We end with a brief discussion of implications for the black hole information problem.

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## 1. Introduction

It is an old and attractive idea that the very massive string states must be black holes because their Schwarzschild radii are bigger than their Compton wavelengths. This identification is most compelling for the special case of extremal supersymmetric black holes: they have vanishing Bekenstein-Hawking entropy and therefore can reasonably be thought of, as string states usually are, as ordinary particles. Sen [1] has found a family of such solutions to the six dimensionally compactified low energy heterotic string effective action. Their charge/mass relation is identical to that of a fundamental string in its right moving ground state but in an arbitrary left moving state. Like such string states, they preserve two of the four supersymmetries and saturate Bogomolny bounds. As a result, the lowest order mass formulas are protected from corrections which might disturb the correspondence of mass spectra. In [2] this correspondence between Sen's extremal black holes and string states was made explicit at the level of the quantum numbers.

There is, however, a small paradox. On the one hand, the mass of the extremal string states is determined by the charges which in turn are related to the *total* left moving oscillator level $N_L$, and the degeneracy at fixed $N_L$ gives rise to an entropy which increases with increasing $N_L$. On the other hand, the horizon area of the black hole solutions is zero and so is the Bekenstein-Hawking entropy. Sen was able to resolve this discrepancy by showing [3] that the area of the "stretched horizon" [4] gives an entropy which matches the left moving oscillator degeneracy. But, if the degenerate substates of an extremal black hole can be identified with specific string states, it should be possible to use string theory to study the dynamics of these otherwise hidden states. That is the goal of this paper.

We study black holes in lower dimensional compactified target spaces by building them out of extended string like solutions in ten dimensional target space. The well known Dabholkar-Harvey (DH) solutions [5] are of this type, but not sufficiently general for our purposes, so we must first extend our knowledge of these solutions in several directions. To begin with, we construct a new class of solutions, generalizing the DH solution to the case of arbitrary left moving excitation. Just as the DH solution should correspond to the $N_R = 1/2, N_L = 1$ state of a winding heterotic string, so the new solutions correspond to its $N_R = 1/2, N_L > 1$ excitations. They are constructed along the lines of past work on "plane-fronted waves" [6,7] but describe a wider class of stringy excitations than have been explicitly considered before. It has already been remarked that certain lower dimensional black holes can be constructed by compactifying the ten dimensional DH fundamental

string. We will show that Sen's complete set of four dimensional extremal black holes can be obtained by compactifying our excited generalizations of the DH string.

In an investigation of string dynamics we compute the classical large impact parameter scattering of two parallel oscillating strings. The effect of the scattering event on the transverse excitations allows us to read off a "polarization dependent" scattering amplitude. We compare these results to the predictions of the heterotic string for the polarization dependent scattering of the fundamental string states and find exact agreement. This considerably strengthens previous evidence from polarization independent scattering on the one hand for identity of the DH soliton and the heterotic string [8,9] and, on the other hand, for the identity of certain four dimensional extremal black holes and the compactified heterotic string [10].

In addition, we have an identification of the four dimensional black holes with specific compactified states of the excited ($N_L > 0$) heterotic string itself. This raises the expectation that questions of the dynamics within the degenerate state space associated with black hole entropy can then be addressed directly, by using string results. We will comment on this possibility and its relevance to the more general problems of black hole information loss.

The paper is organized as follows: In section two we construct and discuss the solutions that generalize the DH string solution. We also show how to construct black hole solutions of the compactified theory by superposing the excited DH string solutions, and show that the complete set of previously known static black hole solutions can be so obtained. In section three we calculate the classical low velocity, small angle scattering of these string solutions, paying particular attention to polarization dependence. In section four we calculate the string theory amplitudes for the corresponding string states in the same limit, i.e. small angle and velocity, and demonstrate agreement with the classical result. The last section contains a discussion of possible consequences of the black hole/string identification and our conclusions.

## 2. Classical Strings in $d = 10$ and Black Holes in $d < 10$

### *2.1. Static strings*

We will begin by reviewing the ten dimensional fundamental string solution of Dabholkar and Harvey [5], a singular solution of $N = 1$ ten dimensional supergravity that

preserves half of the spacetime supersymmetries. This solution was subsequently generalized in [11,12] to include charge and momentum flowing along the string. The action is

$$S_{10} = \int d^{10}x\sqrt{-G}e^{-\Phi}[R + \nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - 2F^I_{\mu\nu}F^{I\ \mu\nu}]$$

where the gauge fields $F^I_{\mu\nu}$ are in the $U(1)^{16}$ subgroup of $E_8 \times E_8$. For an extended string pointing in the direction $\hat{9}$, the nontrivial spacetime fields of the generalized DH string are†

$$(G_{\alpha\beta}) = e^\Phi \begin{pmatrix} -(1+C) & C \\ C & 1-C \end{pmatrix}$$

$$B_{09} = -e^\Phi + 1 \qquad\qquad A^I_0 = -A^I_9 = N^I e^\Phi \tag{2.1}$$

where $\alpha, \beta \in (0,9)$

$$C = R + 2e^\Phi N^I N^I \tag{2.2}$$

and $e^{-\Phi}, R, N^I$ are harmonic functions of the eight transverse coordinates (e.g. $\partial_i\partial_i e^{-\Phi} = 0, \quad i = 1,\ldots,8$). For a string of mass, momentum, and sixteen $U(1)$ charges per unit length equal to $m, p, q^I_L$ these functions have the form

$$e^{-\Phi} = 1 + 2m\Lambda \qquad\qquad R = -2p\Lambda \qquad\qquad N^I = \frac{q^I_L}{\sqrt{2}}\ \Lambda \tag{2.3}$$

with

$$\Lambda = \frac{8}{\pi^3 r^6} \equiv \frac{c_9}{r^6} \qquad\qquad r^2 = x_1^2 + \cdots x_8^2. \tag{2.4}$$

As is usual with supersymmetric solutions, there are multiple string solutions, where the harmonic functions become superpositions, e.g.

$$m\Lambda = \sum_a \frac{m_a c_9}{|\vec{r} - \vec{r}_a|^6}$$

with the $a$–th string located at transverse position $\vec{r}_a$.

It has always seemed more than plausible that these solutions should be identified with states of the fundamental string carrying various amounts of zero-mode momentum. On the other hand, in string theory these momenta must satisfy the $L_0 - \tilde{L}_0 = 0$ level-matching condition, while no conditions are imposed on the corresponding parameters

† We use conventions for the fields and ten dimensional action as in [13]. For the constants we take $\alpha' = 2$ and put the asymptotic value of the string coupling constant to be $g = 1$, so we have $G_N^{2/(d-2)} = 2$ for the Newton constant.

$m, p, q_L^I$ of the classical solution. We think we see how to resolve this mystery: Although the classical solutions are singular at $r = 0$ and are not solutions there in the strict sense unless a source is provided at the singularity, one can make the singularity invisible to outside observers by imposing one condition on the momenta. To see this, consider a light ray moving towards the string. Its trajectory is described at small $r$ by

$$0 = [-w + \mathcal{O}(\frac{1}{\Lambda})]dt^2 + 2w dx^9 dt + [-w + \mathcal{O}(\frac{1}{\Lambda})]dx^9 dx^9 + dx^i dx^i$$

where $w = w(m, p, q_L^I) = -p/m + q_L^I q_L^I/(4m^2)$ is a constant and $\Lambda$ blows up as $r^{-6}$ as $r \to 0$. Thus

$$\left|\frac{dx^i}{dt}\right| = \sqrt{w(m, p, q_L^I)\left(1 - \frac{dx^9}{dt}\right)^2 + \mathcal{O}\left(\frac{1}{\Lambda}\right)}$$

and the time taken for a light ray to escape from $r = 0$ to a finite distance diverges if and only if

$$w(m, p, q_L^I) = 0 \qquad \text{or} \qquad 4mp = q_L^I q_L^I \tag{2.5}$$

We will see later that this is identical to the level matching condition for the fundamental string with corresponding zero-mode momenta and with oscillator excitations $N_R = 1/2, N_L = 1$. One might have expected a correspondence with the strict string ground state, but it is not too surprising to see a typical normal-ordering shift. When we construct classical solutions with excitations corresponding to strings with $N_L \gg 1$ we will again find that the condition for the singularity to be invisible is the same as the string level-matching condition. By itself, this seems to us a significant piece of evidence for the identity of the string solutions with the fundamental string.

### *2.2. Oscillating strings*

Lower dimensional black holes with entropy have quantum numbers which are conjectured to correspond to string states with arbitrary left moving oscillator excitation. This motivates us to generalize the static string solutions of the previous section to solutions carrying propagating transverse waves. In fact, we will be able to construct solutions that represent multiple parallel oscillating strings, each string carrying a different traveling wave.

We start by constructing the explicit solution for one oscillating string. It was not apparent to us how to generalize the static string solutions so we started instead with the chiral null models considered in [6] and elsewhere. We will soon see that a particular chiral

null model gives a generalization of the static string which has transverse excitations. In the natural light-cone coordinates $u = x^9 - x^0,\ v = x^9 + x^0$ the particular solution of interest to us takes the form

$$\begin{aligned} ds^2 =& e^{\Phi(r)} du[dv + K(r)du + 2f'^i(u)dx^i] + dx^i dx^i \\ B_{uv} =& e^{\Phi(r)} \qquad\qquad B_{ui} = 2e^{\Phi(r)} f'^i(u) \end{aligned} \tag{2.6}$$

where $f^i(u)$ are arbitrary functions describing a traveling wave on the string, $e^{-\Phi}$ and $K$ are harmonic functions, and in ten dimensions $e^{-\Phi(r)} = 1 + 2mc_9/r^6$ and $K(r) = 2pc_9/r^6$. Since this metric is not manifestly asymptotically flat, we prefer to make the simple change of coordinates

$$x^i = y^i - f^i(u) \qquad\qquad v = \tilde{v} + \int^u [f'^i(u_0)]^2 du_0 \tag{2.7}$$

which puts the fields in the form

$$\begin{aligned} ds^2 =& e^{\Phi(\vec{r},u)} du \Big[ d\tilde{v} - 2(e^{-\Phi(\vec{r},u)} - 1) f'^i(u) dy^i + \\ & + \Big( (e^{-\Phi(\vec{r},u)} - 1)[f'^i(u)]^2 + K(\vec{r},u) \Big) du \Big] + dy^i dy^i \\ B_{uv} =& e^{\Phi(\vec{r},u)} \qquad\qquad B_{ui} = 2e^{\Phi(\vec{r},u)} f'^i(u) \end{aligned} \tag{2.8}$$

where $\Phi(\vec{r},u) = \Phi(\vec{r} - \vec{f}(u))$ and $K(\vec{r},u) = K(\vec{r} - \vec{f}(u))$. The metric is now manifestly asymptotically flat, and, in the limit $f^i(u) \to 0$, it reduces to the static solution (2.1) with zero charge.

Now we turn to the problem of constructing multiple oscillating string solutions. It turns out that the following simple generalization of the structure of the fields of the single string gives us an ansatz of sufficient generality:

$$\begin{aligned} ds^2 =& F(\vec{r},u) du [dv + K(\vec{r},u) du + 2V_i(\vec{r},u) dx^i] + dx^i dx^i \\ B_{uv} =& F(\vec{r},u) \qquad\qquad B_{ui} = 2F(\vec{r},u) V_i(\vec{r},u) \qquad\qquad \Phi = \Phi(\vec{r},u)\ . \end{aligned} \tag{2.9}$$

All that we have really done is to generalize the chiral null models of [6] by allowing $u$ dependence in $F$. We first demand that this configuration, regarded as a background of heterotic string theory, preserve half of the spacetime supersymmetries. The supersymmetry variations of the fermionic fields are

$$\begin{aligned} \delta_\epsilon \lambda =& \frac{\sqrt{2}}{4\kappa}[-\frac{1}{2}\gamma^\mu \partial_\mu \Phi + \frac{1}{12} H_{\mu\nu\rho}\gamma^{\mu\nu\rho}]\epsilon \\ \delta_\epsilon \psi_\mu =& \frac{1}{\kappa}[\partial_\mu + \frac{1}{4}(\omega_\mu^{\hat{\nu}\hat{\rho}} - \frac{1}{2} H_\mu{}^{\hat{\nu}\hat{\rho}})\Gamma_{\hat{\nu}\hat{\rho}}]\epsilon \\ \delta_\epsilon \chi =& -\frac{1}{4g} F_{\mu\nu}\Gamma^{\mu\nu}\epsilon \end{aligned} \tag{2.10}$$

where $\kappa = \sqrt{8\pi G_N}$ is the gravitational coupling constant, plain greek letters label coordinate indices, and letters with a hat label tangent space indices. Coordinate and tangent indices are related by the zehnbeins $e_\mu^{\hat\nu}$ and $\omega_\mu^{\hat\nu\hat\rho}$ is the corresponding spin connection. $\Gamma_{\hat\nu}$ are the flat space gamma matrices satisfying $\{\Gamma_{\hat\nu}, \Gamma_{\hat\rho}\} = 2\eta_{\hat\nu\hat\rho}$, $\gamma^\mu = e_{\hat\nu}^\mu \Gamma^{\hat\nu}$ and $\gamma^{\mu_1\cdots\mu_n}$ is the antisymmetrized product with unit weight (i.e. dividing by the number of terms). Choosing the zehnbein

$$e_\mu^{\ \hat\nu} = \begin{pmatrix} F^{1/2} & 0 & 0 \\ -F^{3/2}V^2 & F^{1/2} & FV_i \\ 0 & 0 & 1_8 \end{pmatrix}$$

and demanding that the variations (2.10) vanish we find

$$\begin{aligned} &\log F(\vec r, u) = \Phi(\vec r, u) + z(u) \\ &\epsilon = F^{1/4}\epsilon_0, \qquad\qquad \Gamma_{\hat v}\epsilon_0 = 0 \end{aligned} \tag{2.11}$$

where $\epsilon_0$ is a constant spinor and $z(u)$ is an arbitrary function.

The next step is to impose the equations of motion

$$R^{[-]}_{\mu\nu} + D^{[-]}_\mu D^{[-]}_\nu \Phi = 0$$

The notation indicates that the curvature and covariant derivatives are constructed out of the generalized connection $\Gamma^{[-]\mu}_{\nu\delta} = \Gamma^\mu_{\nu\delta} - \frac{1}{2}H^\mu_{\nu\delta}$. These equations summarize the beta functions for both the metric and the antisymmetric tensor. They yield the following additional conditions:

$$\begin{aligned} &\partial_i\partial_i e^{-\Phi} = 0 \\ &\partial_j\partial_j V_i - \partial_i\partial_j V_j + e^{-z}\partial_i\partial_u e^{-\Phi} = 0 \\ &\partial_u\partial_i V_i - \partial_u(e^{-z}\partial_u e^{-\Phi}) - \frac{1}{2}\partial_i\partial_i K = 0 \end{aligned} \tag{2.12}$$

These Laplacian equations can easily be solved and the solution of interest to us is

$$\begin{aligned} e^{-\Phi} &= 1 + \sum_a 2m_a \Lambda(\vec r - \vec f_a(u)) \\ V_i &= -\sum_a 2m_a f_a'^i(u)\Lambda(\vec r - \vec f_a(u)) \\ K &= \sum_a 2p_a(u)\Lambda(\vec r - \vec f_a(u)) \\ z &= 0 \qquad\qquad \Lambda(\vec r) = c_9/|\vec r|^6 \end{aligned} \tag{2.13}$$

where $f^i_a(u)$ is an arbitrary function. This clearly represents a collection of oscillating strings with an arbitrary traveling wave, specified by $f^i_a(u)$, on the $a$–th string. What is surprising and remarkable is that this non-static solution preserves half the supersymmetries and is therefore presumably a BPS-saturated state. Note that this construction of oscillating strings works equally well for any dimension $d > 4$, with $\Lambda = c_{d-1}/|\vec{r}|^{d-4}$.

In string theory, the left moving oscillators associated with the gauge degrees of freedom can also be excited and we would like to find a corresponding classical solution. It turns out that we can find a supersymmetric solution having an oscillating current by adding to (2.13) and (2.9) a set of $U(1)$ gauge fields defined as in (2.1) with

$$N^I(\vec{r}, u) = \sum_a \frac{q^I_{L,a}(u)}{\sqrt{2}} \Lambda(\vec{r} - \vec{f}_a(u)) \tag{2.14}$$

and by augmenting the function $K$ as follows:

$$K(\vec{r}, u) = \sum_a 2p_a(u)\Lambda(\vec{r} - \vec{f}_a(u)) - 2e^{\Phi(\vec{r},u)} N^I(\vec{r}, u) N^I(\vec{r}, u) \tag{2.15}$$

$K$ thus takes the same form as does $[-C]$ in (2.1), now with $u$ dependent $p$ and $q^I_{L,a}$

$$\begin{aligned} p_a(u) &= p_a + \tilde{p}_a(u) \\ q^I_{L,a}(u) &= q^I_{L,a} + \tilde{q}^I_{L,a}(u) \end{aligned}$$

In these equations the tildes denote the oscillating parts while the $p_a, q^I_{L,a}$ denote the constant, zero mode pieces.

With the gauge fields given by (2.14), the function $K$ as in (2.15), and the dilaton and $V_i$'s as in (2.13), we see that we have achieved a multiple string generalization of the previously known static string. This new solution has oscillations in the transverse directions, and oscillating densities for the longitudinal momentum, charge and current.

There remains the question whether these solutions correspond to exact conformal field theories. In [6], Horowitz and Tseytlin demonstrate that what amounts to the single string version of our solution is conformal to all orders in $\alpha'$. In the appendix we present a slight generalization of their argument which, we believe, extends their proof to our multiple oscillating string solution.

Just as for the single static string, there is the question whether one can restrict the parameters so as to make the singularities at the center of each string invisible to outside observers. We will discuss this issue using the same strategy we applied to the single static

string: the trajectory of a light ray moving perpendicular to an oscillating string near its singularity is governed by the equation

$$\left(\frac{dx^i}{dt}\right)^2 + 2\frac{dx^i}{dt}f'^i\left(1-\frac{dx^9}{dt}\right) - w\left(1-\frac{dx^9}{dt}\right)^2 - \mathcal{O}\left(\frac{1}{\Lambda}\right) = 0$$

where $w = -p(u)/m + q_L^I(u)q_L^I(u)/(4m^2)$. The $(t,\vec{r})$ cross term in the trajectory equation comes from the functions $V_i$ and is not present for the static string. Rewriting this as

$$\left|\frac{d}{dt}[\vec{r}-\vec{f}(u)]\right| = \sqrt{[w+(\vec{f}')^2]\left(1-\frac{dx^9}{dt}\right)^2 + \mathcal{O}\left(\frac{1}{\Lambda(\vec{r}-\vec{f}(u))}\right)}$$

we see that it takes a light ray an infinite time to escape from the singularity, thereby indicating that the singularity is hidden, if and only if the condition

$$4p(u)m = [q_L^I(u)q_L^I(u)] + 4m^2[f'^i(u)f'^i(u)] \tag{2.16}$$

is satisfied. We will see in the next section that this "invisible singularity" condition agrees, modulo the normal-ordering subtlety, with the $L_0 = \tilde{L}_0$ stringy level matching condition with left moving oscillators excited.

As a final check on our understanding of the physics of these solutions, we should verify that the excited strings do indeed transport physical momentum and angular momentum. Since we have written the metric in a gauge where it approaches the Minkowski metric at spatial infinity, we can use standard ADM or Bondi mass techniques to read off kinetic quantities from surface integrals over the deviations of the metric from Minkowski form. Following [5], we pass to the physical (Einstein) metric $g_E = e^{-\Phi/4}g_{string}$, expand it at infinity as $g_{E\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and use standard methods to construct conserved quantities from surface integrals linear in $h_{\mu\nu}$. Of course the Bondi mass and its kinetic analogs are most appropriate for this problem since things in general depend on a light-cone coordinate. In particular, for a single oscillating string, we find that the transverse momentum on a slice of constant $u$ is

$$P_i = mf'^i(u) \tag{2.17}$$

in precise accord with "violin string" intuition about the kinematics of disturbances on strings. Similarly, the net longitudinal/time momentum $\Theta_{\alpha\beta}$ in a constant $u$ slice is

$$(\Theta_{\alpha\beta}) = \begin{pmatrix} m+P_u & -P_u \\ -P_u & -m+P_u \end{pmatrix}$$

where

$$P_u = p(u) - 2mf'^i(u)f'^i(u) \ .$$

is the physical longitudinal momentum per unit length. In this expression we have used the fact that $P_v = m/2$. Finally, we consider angular momenta. For the string in ten dimensions there are four independent (spatial) planes and thus four independent angular momenta $M^{ij}$. Evaluating as an example $M^{12}$ we obtain

$$M^{12} = m(f'^1 f^2 - f'^2 f^1)(u) \ .$$

There are no surprises here, just a useful consistency check.

*2.3. $d \leq 9$ Black Holes From $d = 10$ Fundamental Strings*

The static multi-string solutions discussed at the beginning of this section can be used to produce pointlike solutions of the toroidally compactified theory in lower dimensions: all that is needed is to place the centers of the strings on a lattice in the transverse space and to compactify the $\hat{9}$ direction on a circle. To be more precise, we build a periodic $(9-d)$ dimensional array of strings in the $\hat{d}$ to $\hat{8}$ directions by taking $\Lambda_d = \sum_{Lattice} \Lambda(\vec{r} - \vec{r}_a)$. For large $(d-1)$ dimensional spatial distances $\rho$ we can ignore the dependence on the internal dimensions and find, to leading order in large $\rho$,

$$\Lambda_d = \frac{c_d}{\rho^{d-3}} \qquad \text{where} \quad c_d = \frac{16\pi}{[(d-3)\omega_{d-2}]} \tag{2.18}$$

and $\omega_d$ is the area of the $d$ dimensional unit sphere. We could have taken directly $\Lambda = c_d/\rho^{d-3}$ as a solution of the Laplace equation in the uncompactified dimensions, but this obscures the essential connection to underlying string states. As we will now show, the result of this proceedure can be interpreted as a lower-dimensional extremal black hole. The general idea that ten-dimensional string solutions can be used to generate four-dimensional black holes is not new and has been explored in [14,15,16,17]. Our contribution will be a more precise understanding of the relation between black hole parameters and string quantum numbers.

We now look in more detail at the $d$-dimensional fields generated by this compactification. Using the dimensional reduction procedure of [18] and the conventions of [13], we find that the $d$-dimensional dilaton is

$$e^{-2\Phi_d} = e^{-2\Phi_{10}}(G_{99}) = e^{-\Phi_{10}}(1-C) = 1 + 2(m+p)\Lambda_d + (4mp - q_L^2)\Lambda_d^2 \ . \tag{2.19}$$

This and all the other fields obtained by dimensional reduction turn out to be identical to those of Sen's four-dimensional black holes and their higher-dimensional generalizations [1,19] with the parameter identifications

$$\begin{aligned} M_{ADM} = &\ (m+p) \\ [\frac{\omega_{d-2}}{4\pi}]\frac{1}{2\sqrt{2}}Q_R^a = &\ (m+p)\delta^{a1} \\ [\frac{\omega_{d-2}}{4\pi}]\frac{1}{2\sqrt{2}}Q_L^a = &\ (m-p)\delta^{a1}+q_L\delta^{aI} \end{aligned} \tag{2.20}$$

where $a$ is any index from $1,\ldots(10-d)$ and $I$ is any index from $(11-d)\ldots(26-d)$, and both $a$ and $I$ are $U(1)$ group labels. These are not completely generic charges since the first six components of $Q_L$ are pointing in the same direction as $Q_R$. A black hole with generic charges can be obtained by boosting the previous solution (2.1) along an internal direction and then reducing back to $d$ dimensions.

We noted earlier that in order to hide the singularity of the static string solution, the densities $m$, $p$ and $q_L$ had to satisfy the level-matching constraint (2.5). In terms of the parameters of the lower-dimensional black holes (2.20), this condition reads

$$\vec{Q}_R^2 = \vec{Q}_L^2 \ .$$

From the field theory point of view there is, however, nothing wrong with the lower-dimensional black holes of [1,19] which do not satisfy this condition: they are still perfectly supersymmetric and extremal. Moreover, these black holes have entropy and explanation of this entropy was one of the main goals of our work. It is a rather natural guess that the missing black holes arise from toroidally compactifying strings which are in the right moving ground state and hence annihilated by half the supersymmetries, but in an excited state of the left moving oscillators. Such string states satisfy the level-matching condition

$$p_R^2 - p_L^2 = 2(N_L - 1) \tag{2.21}$$

and ought to yield, upon compactification, black holes with $\vec{Q}_R^2 \neq \vec{Q}_L^2$.

This suggests that we study the lower dimensional black holes arising from compactification of the oscillating string solutions (2.13). More precisely, those solutions presumably correspond to string states where the oscillators are in a coherent state with macroscopic expectation value for $N_L$. One slight puzzle is that these string solutions oscillate, while

the black hole solutions are static. Of course, all fields are periodic in $u = x^9 - x^0$, because $x^9$ is compactified, and, for small compactification radius, one might argue that the oscillations should "average out", leaving an effective static solution. We can make this argument more precise: We can superimpose any collection of solutions of the type (2.13), in particular a collection of oscillating solutions which differ in the phase of oscillation but are otherwise identical. Mathematically this corresponds to replacing $u \to u + u_0$ and integrating over $u_0$. This is a well defined "averaging" procedure that is guaranteed to give a supersymmetric, static solution.

When we further compactify to $d$ dimensions, if the oscillations point in the internal dimensions we do indeed obtain Sen's most general black hole solutions [19,3]. In addition, the degrees of freedom giving rise to the entropy of the black holes are very clear in our construction: they are precisely those of the ten dimensional string. When the oscillators point in the external uncompactified directions we expect to get rotating black holes. Let us now review the import of the invisible singularity condition for the oscillating string which gives rise to the static (nonrotating) black holes. After integrating the ten dimensional fields over $u$ and expressing the charges and momenta in terms of the $Q_{L,R}$ of the lower-dimensional black hole, the condition (2.16) becomes

$$\frac{1}{8}[\frac{\omega_{d-2}}{4\pi}]^2(\vec{Q}_R^2 - \vec{Q}_L^2) = 2m^2\langle(\vec{f}')^2\rangle + \frac{1}{2}\langle(\tilde{q}_L^I)^2\rangle$$

This, together with (2.20)and (2.21), leads to the natural interpretation that the left moving oscillator level for the string state corresponding to the static black hole in $d$ dimensions is related to the mean square oscillations in ten dimensions by

$$N_L = m^2\langle(\vec{f}')^2\rangle + \frac{1}{4}\langle(\tilde{q}_L^I)^2\rangle \tag{2.22}$$

(quantum normal ordering effects, to which this classical computation should not be sensitive, would replace $N_L$ by $N_L - 1$). Looking at (2.22), we see that the left moving oscillator level comes from both macroscopic oscillations associated to the spacetime directions and charge oscillations associated to the internal directions. In the next section, we will see that the same condition arises from quantum level matching.

## 3. Classical Scattering of Fundamental Strings

In this section we will study the classical scattering of fundamental strings in ten dimensions. From this calculation we will also extract information about the scattering of objects in lower dimensions that can be constructed from the fundamental string. This includes all extremal electrically charged black holes in the compactified theory. The calculation of the scattering will be done in the low velocity and small angle approximation. The level of approximation used will be the "test string" approximation, where we consider one string moving in the background of the other. This approximation is well justified if the scattering angle is small, since it amounts to a neglect of the recoil of the background string.

### *3.1. Scattering of generalized fundamental strings in $d = 10$.*

We consider a background string as in (2.1) with parameters $m, p$ and $q$. Again, we take the gauge fields to be in a $U(1)^{16}$ subgroup of $E_8 \times E_8$. It will be convenient to bosonize the 32 left moving fermions of the heterotic string in terms of 16 bosons $\varphi^I$, $I = 1, \ldots, 16$. To obtain the worldsheet action in terms of the left moving bosons and the right moving fermions, we start with the standard worldsheet action written in terms of left and right fermions. After bosonization, as is usual, anomaly terms which would arise at one loop for the chiral fermions arise at the classical level for the bosons. One also finds that the antisymmetric tensor must transform under gauge transformations. The precise form of the worldsheet action is found by demanding gauge invariance and supersymmetry. We find that

$$\begin{aligned} S = &-\frac{1}{2\pi}\int d\tau d\sigma(\sqrt{-h}h^{mn} + \epsilon^{mn})\left\{\partial_m X^\mu \partial_n X^\nu (G_{\mu\nu} + 2A^I_\mu A^I_\nu + B_{\mu\nu}) + \right. \\ &\left. +\partial_m\varphi^I\partial_n\varphi^I - 2\sqrt{2}\partial_m X^\mu A^I_\mu \partial_n\varphi^I + \frac{i}{2}\psi^\mu\gamma_m\mathcal{D}_n\psi^\nu G_{\mu\nu}\right\} \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} \mathcal{D}^\mu_{m\nu} =& \delta^\mu_\nu\partial_m + (\Gamma^\mu_{\delta\nu} + \frac{1}{2}H^\mu_{\delta\nu})\partial_m X^\delta + \sqrt{2}F^{I\mu}{}_\nu(\partial_m\varphi^I - \sqrt{2}A^I_\delta\partial_m X^\delta) \\ H_{\mu\nu\rho} =& (\partial_\mu B_{\nu\rho} - 2A^I_\mu F^I_{\nu\rho}) + \quad \text{(cyclic permutations)}\ . \end{aligned} \tag{3.2}$$

It will be important to know the form of the supercurrent constraint:

$$(\sqrt{-h}h^{mn} - \epsilon^{mn})\gamma^r\gamma_n\psi^\mu\partial_r X^\nu G_{\mu\nu}(X) = 0\ . \tag{3.3}$$

Our aim here is to generalize the results of [9] to include the response of fermionic and oscillator degrees of freedom carried by the string to small angle scattering. We first make the gauge choice

$$h_{\tau\sigma} = 0 \qquad\qquad h_{\tau\tau} = h_{\tau\tau}(\tau) \qquad\qquad h_{\sigma\sigma} = h_{\sigma\sigma}(\tau) \ .$$

Our general strategy, following [9], is to reduce the worldsheet (string) action an effective worldline (particle) action by choosing a restrictive ansatz for the worldsheet fields which reflects the fact that most degrees of freedom do not actively participate in a low velocity scattering event. A sufficiently general field configuration is

$$\begin{aligned}
X^0 =& X^0(\tau) + X^0_L(\sigma^-, \tau) \\
X^i =& X^i(\tau) + X^i_L(\sigma^-, \tau) \\
X^9 =& m'\sigma + X^9(\tau) + X^9_L(\sigma^-, \tau) \\
\varphi^I =& \frac{q^I}{2}\sigma^- + \varphi^I_L(\sigma^-, \tau) \\
\psi^\mu =& \psi^\mu(\tau)
\end{aligned} \tag{3.4}$$

where $\sigma^- = \int^\tau e d\tau - \sigma$ with $e = \sqrt{-h_{\tau\tau}/h_{\sigma\sigma}}$. The $X^\mu(\tau)$ describe the motion of the string center of mass. The functions $X^\mu_L, \varphi^I_L$ describe the state of the left moving oscillators. They are periodic functions of $\sigma^-$ and we take them to be slowly varying functions of the second argument $\tau$ in order to allow for possible adiabatic changes in the state of left-moving oscillation in the course of a soft collision. In what follows, we assume that this variation is negligible and will later verify that this is self-consistent. The right moving fermion $\psi$ is taken to be a worldsheet zero mode (i.e. a function of $\tau$ only) in order to focus on the dynamics of BPS-saturated states. Again, we have to verify that higher fermionic modes are not excited during a collision.

The left moving oscillators influence the center of mass motion only through the stress energy constraint, a helpful simplification. The stress energy tensor would classically be set equal to zero, but, as shown in [9], to properly account for quantum zero point energies and, more subtly, the difference between left and right moving sectors in the heterotic string, one must instead set $T_{mn} = \begin{pmatrix} e^2/2 & -e/2 \\ -e/2 & 1/2 \end{pmatrix}$. This constraint is more conveniently recast as

$$\widetilde{T}_{mn} = \frac{(N_L - 1)}{2} \begin{pmatrix} -e^2 & e \\ e & -1 \end{pmatrix} \ , \tag{3.5}$$

where $\widetilde{T}_{mn}$ is the limit of the energy momentum tensor when the left moving oscillators $X_L^\mu$ are set to zero and

$$N_L = 2\int \frac{d\sigma}{\pi}\left(\partial_- X_L^\mu \partial_- X_L^\nu G_{\mu\nu} + \partial_-\varphi_L^I \partial_-\varphi_L^I\right)$$

is what would normally be identified as the left-moving oscillator level. For an oscillating string of the kind studied in the previous section, the expression for $N_L$ reduces to (2.22), as promised. Note that (3.5) amounts to imposing only the zero mode component of the Virasoro constraints, which is what is necessary now that we are ignoring the left moving oscillators in the action.

Under the above assumptions, we can reduce the worldsheet action (3.1) to a worldline action for the center of mass coordinates of the string. The essential point is that, after integrating over $\sigma$ and applying the energy momentum constraint (3.5), the contribution of the left-moving oscillators to the action collapses to $-(N_L-1)e$ (where we include the quantum normal-ordering constant). The resulting effective worldline action is

$$\begin{aligned} S = -\frac{1}{2}\int d\tau \Big\{ & -e^{-1}\dot{X}^2 + eX'^2 - (N_L-1)e + 2\dot{X}^\mu X'^9 B_{\mu 9} + \\ & +2\sqrt{2}\partial_-\varphi^I(\dot{X}^\mu + eX'^\mu)A_\mu^I - \frac{i}{2}e^{-1/2}\bar{\psi}^\mu(\mathcal{D}_0 - e\mathcal{D}_1)(\gamma_{\hat{0}}+\gamma_{\hat{1}})\psi^\nu G_{\mu\nu}\Big\}\ . \end{aligned} \tag{3.6}$$

The oscillator level $N_L$ will be a constant of the motion and strings with left-moving excitations will be just as easy to deal with as ground state strings. The term with $\mathcal{D}_1$ contributes only through the term in the connection involving $X'^9$. We have also found it convenient to make the redefinition $\psi \to (-h)^{1/8}\psi$.

Now let us think about solving this action for the motion of one string in the background of another. We will first work in the crude leading approximation adequate to study the motion of the center of mass coordinates. Later we will worry about the more subtle effects that give polarization-dependent terms in the scattering. As the Lagrangian is independent of $X^9$, we can eliminate $X^9$ from the action in favor of a conserved momentum $p' \equiv P_9$ which will include some fermionic contributions. The value of $p'$ is determined by the off-diagonal part of (3.5). Next, we use the equation of motion for $e$ to eliminate $e$ from the action (3.6) and the supercurrent (3.3). In addition we choose the gauge $X^0(\tau)=\tau$. We then expand the action to second order in the velocity for the bosonic terms, to first order in velocity for the fermionic terms and keep only terms quadratic in the fermions. With this, the supercurrent constraint becomes

$$\psi_0 + \psi_9 + v^i\psi^i = 0 \qquad \text{where} \qquad v^i = \dot{X}^i(\tau)\ .$$

We can use it to replace $\psi_0$ in the action, eliminating $\psi_9$ as well. We finally obtain an action for unconstrained center of mass coordinates and their associated right moving fermions:

$$\begin{aligned} S =& \int d\tau \left\{ \frac{1}{2} g v^2 - \frac{i}{2} \left\{ g \psi^i \partial_0 \psi^i + \psi^i \psi^j (v^i \partial_j g - v^j \partial_i g) \right\} \right\} \\ g =& M' + (2m'p + 2mp' - q^I q'^I) \Lambda \end{aligned} \tag{3.7}$$

where $M' = m' + p'$, $\Lambda$ is as in (2.4), and the primed and unprimed quantities refer to the test string and the background string respectively. The function $g$ is (proportional to) the metric on moduli space and governs the low-energy scattering in the usual way. It depends explicitly on the charge and momentum parameters of the strings and implicitly on the left-moving oscillator level through the level-matching constraints. By a slight generalization, we can derive the worldline action for a string moving in the background of an arbitrary superposition of fundamental strings. There is an asymmetry in (3.7) between the two strings which is due to the fact that we fixed the position of the background string. A symmetric expression can be found by demanding that it be symmetric in the two strings, invariant under translations, and that it reduces to (3.7) when we fix the position of one of the two strings. For the bosonic part of the action, this prescription gives

$$S = \int \frac{1}{2} M v_1^2 + \frac{1}{2} M' v_2^2 + (2m'p + 2mp' - q^I q'^I) \Lambda(r_1 - r_2) \frac{1}{2} (v_1 - v_2)^2 \tag{3.8}$$

For the moment we will ignore the fermionic terms in (3.7), and calculate the small angle scattering cross section due to the metric on moduli space. We could infer it from the classical relation between scattering angle and impact parameter or we could quantize the Lagrangian (3.8) and compute the Born approximation. The two results are different because quantum effects make the classical limit unreliable for small scattering angles in dimensions $d \geq 5$: the uncertainty principle prevents the relative fluctuations in scattering angle and impact parameter from being simultaneously small. The Born approximation is, however, reliable in all dimensions when applied to small angle low momentum transfer scattering. We apply it to (3.8) by taking the $r$-dependent part (the part proportional to $\Lambda$) as the perturbation and find

$$\frac{d\sigma}{d\Omega} \sim \frac{\Delta_{12}^2}{\theta^4}$$

where $\Delta_{12} = 2m'p + 2mp' - q^I q'^I$. The feature that the cross-section depends on angle but not on energy is a typical metric on moduli space result.

The result agrees with previous calculations of string-string scattering, such as [9], and generalizes them by evaluating the metric on moduli space for the most general string in its right moving ground state. In this regard, it may be helpful to use the level-matching conditions to rewrite $\Delta_{12}$ in terms of the left moving oscillator levels:

$$\Delta_{12} = \frac{1}{2}[(N_L - 1) + (N_L' - 1) + (q_L - q_L')^2]$$

This makes it clear that for identical $N_L > 1$ strings, the Born approximation cross section goes as

$$\frac{d\sigma}{d\Omega}_{\{N_L>1\}} \sim \frac{(N_L-1)^2}{\theta^4} \; . \tag{3.9}$$

When $N_L = 1$, as is true for the fundamental string solution of [5] or the "$a = \sqrt{3}$" extremal charged dilaton black hole, the metric on moduli space is flat and the worldline action must be expanded to one more order in $v^2$ to extract the cross section. When do such an expansion, we find the improved action

$$S_{\{N_L=1\}} = \frac{1}{2}\int d\tau\{Mv^2 + \frac{M}{4}v^4(1 + 2M\Lambda + q^I q^I \Lambda^2)\} \tag{3.10}$$

(where $M = m + p$ and $4mp = q^I q^I$ because of the $N_L = 1$ condition) and a Born cross section

$$\frac{d\sigma}{d\Omega}_{\{N_L=1\}} \sim \frac{M^2 v^4}{\theta^4} \tag{3.11}$$

Doubts had been expressed in the literature on whether radiation terms might not be of comparable order to the $O(v^4)$ potential terms responsible for the scattering. We estimate the effective forces due to radiation in an identical particle system to be of still higher order than $v^4$, hence negligible. Indeed, it is shown in [20] that in the case of identical particles interacting through gravity one can find a reliable two-body Lagrangian to order $v^4$.

*3.2. Polarization dependence of the scattering.*

We now want to go a step beyond the metric on moduli space approximation and consider the effect of scattering on the "polarization" state of the strings. The heterotic string has both right moving and left moving polarization degrees of freedom. The right moving ones are described by the fermions $\psi^i$ that have already appeared in the action (3.7) and it is quite easy to discuss their scattering dynamics. The left movers are more subtle and we will discuss their behavior separately.

The right moving degrees of freedom always remain microscopic, in the sense that, even for large mass strings, their excitation number remains finite and there are no long range fields associated with them. They are analogous to the spin degrees of freedom of a massive particle. To describe their dynamics, we just have to quantize the full Lagrangian (3.7), including the $\psi^i$. After the rescaling $\psi \to g^{-1/2}\psi$, the $\psi^i$ have the standard anticommutation relations of $\gamma$ matrices. The Hilbert space is spanned by eight-dimensional spinors and describes the Ramond sector of the heterotic string, as we can also see from (3.4). To get the right physics, we have to project onto a definite $S0(8)$ chirality of the spinors. In the Neveu Schwarz sector, the fermions are antiperiodic in the $\sigma$ direction, and we change the ansatz in (3.4) for picking out the fermion ground state to $\psi^\mu(\tau,\sigma) = (e^{i\sigma}\psi^\mu(\tau) + \text{c.c.})/\sqrt{2}$.

We will now concentrate on the NS sector. The reduction to a worldline action gives something similar to (3.7) but with complex fermions $\psi^i(\tau)$. The NS fermions behave as fermionic creation and annihilation operators and the lowest allowed states have a single creation operator acting on the vacuum (as usual, the vacuum itself is eliminated by the GSO projection). The action becomes

$$
\begin{aligned}
S =& \int d\tau \, (L_{free} + L_{int}) \\
L_{int} =& \frac{1}{2} v^2 \Delta_{12} \Lambda - i \frac{\Delta_{12}}{M'} (v^i \partial_j \Lambda - v^j \partial_i \Lambda) \psi^{i\dagger} \psi^j
\end{aligned}
$$

where the fermions are normalized so that $\{\psi^{i\dagger}, \psi^j\} = \delta_{ij}$ and we have kept in the fermionic term only the leading power of $1/r^6$ since we are interested in small angles. Representing the states by $\eta^i \psi_i^\dagger |0\rangle$ we can calculate the Born amplitude

$$
\mathcal{A} = \eta_1^i \eta_3^i \Delta_{12} \frac{k^2}{2} \mathcal{F}_q(c_9/r^6) + \eta_3^i (q_i k_j - k_j q_i) \eta_1^j \mathcal{F}_q(c_9/r^6) \tag{3.12}
$$

where $\mathcal{F}_q(c_9/r^6)$ is the Fourier transform of $c_9/r^6$ with respect to the transfer momentum $q$. Here $\eta_1, \eta_3$ denote the polarizations of the initial and final states. If the initial and final polarizations are equal we get the same Born amplitude we found in the previous bosonic calculation:

$$
\mathcal{A}_{no\ flip} = \frac{\Delta_{12}}{\theta^2}
$$

In order to make a concrete calculation of polarization changing amplitudes, let us consider two strings with equal masses and charges: one being a background string sitting at the origin and the other string coming toward it with small velocity $\vec{v} = v\hat{2}$ and with a large impact parameter $b$ in the direction $\hat{3}$, as depicted in Fig. 1.

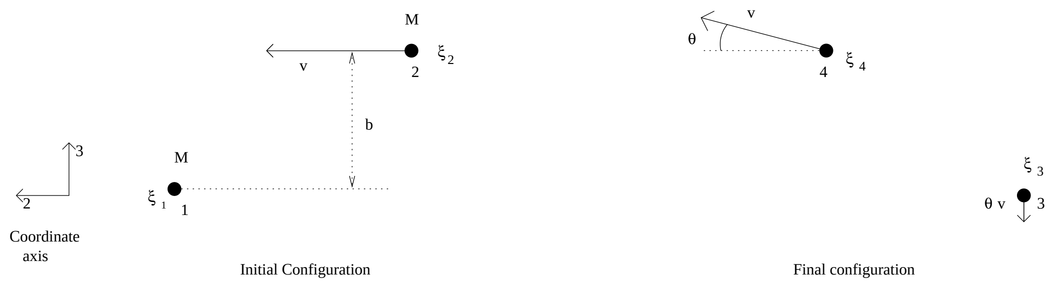


Figure 1: Scattering configuration. One string is initially at rest and the other has velocity $v$ and impact parameter $b$ and is scattered by a small angle $\theta$.

For this situation we have for small angles $\vec{q} = M' v\theta\ \hat{3}$ . The amplitude for a polarization to rotate from the direction $\hat{3}$ to $\hat{2}$ can then be read off from (3.12):

$$\mathcal{A}_{32} = 2\frac{\Delta_{12}}{\theta}$$

The crucial point is that, apart from kinematic factors, the same potential governs both the spin-flip and no-flip Born amplitude. A more concise statement of the result is

$$\frac{\mathcal{A}_{flip}}{\mathcal{A}_{noflip}} = 2\theta \tag{3.13}$$

The probability that a polarization originally in the direction $\hat{3}$ would flip in the course of scattering through angle $\theta$ and wind up pointing in the direction $\hat{2}$ is $P_{flip} = (2\theta)^2$. The calculation is essentially the same for the Ramond case.

At this point we would like to comment on the expectation that, for solitons breaking some of the target space supersymmetries, the full action in moduli space should possess a worldline supersymmetry. The action (3.7) is not supersymmetric. This is not a contradiction because we are treating the Neveu-Schwarz and Ramond sectors on a different footing. The supersymmetries would transform one sector into the other.

We now turn to the question of the left moving polarization dependence. Unlike the right moving polarizations, which are analogous to semiclassical spin degrees of freedom, these polarizations can have a macroscopic effect since the energy in these modes can be comparable with the total mass. Indeed, we saw in previous sections how to construct classical solutions carrying these polarization degrees of freedom. In principle we could obtain a low energy action which includes the left moving oscillators by doing a more accurate derivation of (3.7), much as we did for the fermions. Instead of doing this we will start with the classical equations of motion of the full worldsheet action (3.1) and project

out linear decoupled effective equations of motion for the left moving oscillators. This is only possible in the small-angle scattering limit, of course. It would be nice to have a more unified treatment of the different kinds of polarization degrees of freedom, but our ad hoc approach is adequate for our current purposes.

For the analysis, it is more convenient to choose a coordinate system in which the test string is sitting at the origin and the background string is moving as in Fig. 1 (rather than the other way around). Now we allow the background string to have any nonzero $m, p, q^I$. The equations of motion for the test string in conformal gauge are

$$\begin{aligned} \partial_+\partial_- X^\mu =& -(\Gamma^\mu{}_{\nu\delta} - \frac{1}{2}H^\mu{}_{\nu\delta})\partial_+ X^\nu \partial_- X^\delta + \\ & + \sqrt{2}F^{\mu I}_\nu \partial_+ X^\nu (\partial_-\varphi^I - \sqrt{2}A^I_\delta \partial_- X^\delta) \\ 0 =& \partial_-(\partial_+\varphi^I - \sqrt{2}A^I_\mu \partial_+ X^\mu) \end{aligned} \tag{3.14}$$

where $H$ is as defined in (3.2). We write the coordinates of the string as

$$\begin{aligned} X^0 =& (m' + p')\tau + X^0_L(\sigma^-, \tau) \\ X^9 =& m'\sigma + p'\tau + X^9_L(\sigma^-, \tau) \\ X^i =& X^i_L(\sigma^-, \tau) \\ \varphi^I =& \frac{q^I}{2}\sigma^- + \varphi^I_L(\sigma^-, \tau) \end{aligned} \tag{3.15}$$

where $\sigma^\pm = \tau \pm \sigma$ are the light cone coordinates on the string world sheet. In the above equations, $X_L$ and $\varphi_L$ are periodic functions of $\sigma^-$ and slowly varying functions of the second argument $\tau$. This explicit $\tau$ dependence is very small ($\partial_\tau X^\mu_L << \partial_{\sigma^-} X^\mu_L$) and is caused by the time dependence of the background. We also assume that the oscillation amplitude is small compared to the characteristic scale over which the background fields vary, which can be achieved if the amplitude is much smaller than the distance between the two strings. These approximations enable us to replace in the background fields the average values of the coordinates, so that the equation (3.14) becomes linear in $X^\mu_L$. When we replace (3.15) in (3.14) we get a $\sigma$ independent part which represents the velocity dependent total force on the string. This force is responsible for the no-polarization-flip scattering described above. The sigma-dependent part gives a set of equations for the evolution of the left-moving oscillators:

$$\begin{aligned} \partial_\tau(\partial_- X^\mu_L) = (m' + p') \Big\{ & -[\Gamma^\mu{}_{0\delta} - \frac{1}{2}H^\mu{}_{0\delta} + \Gamma^\mu{}_{9\delta} - \frac{1}{2}H^\mu{}_{9\delta}]\partial_- X^\delta_L + \\ & + \sqrt{2}(F^{\mu I}_0 + F^{\mu I}_9)(\partial_-\varphi^I_L - \sqrt{2}A^I_\delta \partial_- X^\delta) \Big\} \end{aligned} \tag{3.16}$$

and an analogous equation for $\varphi^I_L$. The right hand side of (3.16) vanishes identically for a static background string. When we boost it to get the expression for the moving string the result is no longer zero and can be expanded in powers of the velocity. The overall effect of the background string on the oscillators is a change in the direction of oscillation. This change is independent of the frequency of the oscillator because we can Fourier transform in $\sigma_-$ both sides of (3.16). Replacing in (3.16) the values of the backgound fields for a moving string in (3.16), we find

$$\partial_{X^0}\partial_- X^\mu_L = \frac{v^2}{2}(p^\mu_L \partial_\nu \Lambda - p_{L\nu}\partial^\mu \Lambda)\partial_- X^\nu_L \tag{3.17}$$

where we kept only the leading terms in $1/r^6$ and $p^\mu_L$ is the left moving transverse momentum of the background string.

Since the equations of motion for the left moving oscillators are linear, we can read off the interaction Hamiltonian from (3.17). We introduce creation and annihilation operators for the transverse oscillators that satisfy $[a^{\mu\dagger}_n, a^\nu_m] = \eta^{\mu\nu}\delta_{nm}$. The interaction Hamiltonian then becomes

$$H_{int} = -i\frac{v^2}{2}(p_{L\mu}\partial_\nu \Lambda - p_{L\nu}\partial_\mu \Lambda)\sum_{n>0} a^{\mu\dagger}_n a^\nu_n \tag{3.18}$$

This leads to a Born amplitude

$$\langle \chi_{final}| v^2 \mathcal{F}_q(c_9/r^6)(q_\mu p^\nu_L - q^\nu p_{L\mu})\sum_{n>0} a^{\nu\dagger}_n a^\mu_n |\chi_{initial}\rangle$$

where $\chi_{initial,final}$ are the initial and final oscillator states. They satisfy the usual Virasoro physical state conditions. The evolution dictated by (3.18) preserves them. In fact, it can be seen that the terms proportional to the longitudinal momentum in (3.18) change the polarizations in such a way that the physical state conditions are satisfied. We see that in this approximation at most one oscillator can change its state. In order to take a specific example consider the case in which an oscillator that is originally pointing in the direction $\hat{3}$ flips to the direction $\hat{2}$. The ratio of polarization changing to polarization preserving amplitudes is

$$\frac{\mathcal{A}_{flip}}{\mathcal{A}_{no\ flip}} = \frac{v^2 M^2 \theta}{\Delta_{12}} \tag{3.19}$$

We can now justify an important simplifying assumption introduced at the beginning of this section: We assumed in writing (3.6) that terms leading to changes in the left-moving oscillator state could be neglected, at least in the leading "metric on moduli space"

approximation to the scattering. The calculation we have just completed, showing the vanishing of the polarization flip probability as $v \to 0$, obviously justifies that initial assumption.

We note finally that if we are scattering two *identical* $N_L = 1$ strings this ratio becomes undefined because of the vanishing denominator and a more accurate calculation must be done. We omit the details, but the result is $2\theta$ just as it was for the right moving fermions, as one would expect. All these more or less complicated polarization dependences are not so much interesting for themselves as for the way they will compare with direct string theory calculations of the same quantities.

### *3.3. Lower dimensional objects*

The next thing we would like to do is to investigate scattering involving lower dimensional black holes. To begin with, let us consider two body black hole scattering. We have seen that $d$ dimensional black holes in the compactified theories can be thought of as string arrays in the uncompactified ten dimensional theory. If we scatter two such objects, then we can imagine doing a "test array" approximation, where we consider one array moving in the field of the other. This idea actually works, for two reasons. First of all, each string in the test array moves independently of its array-mates, since there are no static forces between the strings of a given array by virtue of supersymmetry. The Lagrangian for the motion of each of these strings reduces, in the low velocity limit, to (3.7) but with the function $\Lambda$ being that of the other array of strings. The second important point is that there are only two body forces in this Lagrangian, since the interaction term is a sum of the interaction terms of the test string with each of the background strings, and since the Lagrangian (3.7) represents only two body forces. Note that, in general, one might have expected to find higher-body forces; in fact, for four dimensional extremal Reissner-Nördstrom black holes there are up to four body interactions [21].

The Lagrangian (3.7) thus enables us to calculate once and for all the two body scattering cross section for charged extremal black holes in $d$ dimensions, in the low velocity and small angle approximation. Before writing the final Lagrangian, we make a check of the static forces between $d$ dimensional extremal black holes. We find that, in order for the static force to vanish, the right moving charges of both black holes have to be parallel: $Q_R = \lambda Q'_R$ with $\lambda > 0$. Then we have that the low velocity Lagrangian is

$$S = \int \frac{1}{2} v^2 \left[ M' + \left( \frac{\omega_{d-2}}{8\sqrt{2}\pi} \right)^2 \frac{(\vec{Q}_R \cdot \vec{Q}'_R - \vec{Q}_L \cdot \vec{Q}'_L) c_{d-3}}{\rho^{d-3}} \right]$$

for a $d$ dimensional black hole.

This Lagrangian can in fact also be obtained directly in $d$ dimensions by using the test particle approximation, and it seems to us that this agreement is another consistency check of the correspondence we have found between our classical ten dimensional strings and lower dimensional static black holes.

We see from this Lagrangian that the metric on the lower dimensional black hole moduli space is of the same form as for the string in ten dimensions, except for the power of the distance. The cross section is then the same as in ten dimensions, up to an overall constant which is independent of any of the parameters of the black holes.

The metric on moduli space is zero only in the case where we have two identical $N_L = 1$ black holes ($N_L = 0$ black holes have $Q_L > Q_R = 2\sqrt{2}M$ and contain naked singularities). In this case, performing a $O(10-d) \times O(26-d)$ rotation and a subsequent redefinition of the dilaton, we can reduce the system to a pair of $a = \sqrt{3}$ dilaton black holes, whose moduli space metric was shown to be flat [22,10,23]. Using the Lagrangian (3.10) for identical $N_L = 1$ strings, we find the first nonvanishing term in the black hole lagrangian in this case by simply replacing $\Lambda$ in (3.10) with its value for a $d$ dimensional black hole (2.18). The cross section is then again the same as for ten dimensions, up to the same overall constant as for the $N_L > 1$ case.

Another calculation that we wish to do is the scattering of massless particles off a black hole. This is again not so interesting in itself as for later comparison with the string theory answer. To do this calculation classically, we consider small perturbations of these massless fields around the black hole background and expand the action to second order. Obviously, the first order perturbations vanish because the black hole background is a solution of the equations of motion, so we have that

$$S[\phi_0^i + \delta\phi^i] \sim S[\phi_0^i] + \frac{1}{2}\frac{\delta^2 S}{\delta\phi^i\delta\phi^j}\delta\phi^i\delta\phi^j$$

where $\phi^i$ denotes all the fields appearing in the full $d$ dimensional action. Using the $O(10-d)\times O(26-d)$ invariance of the $d$ dimensional action and the asymptotic conditions [1] on the fields, we can assume that the right and left moving charges are parallel and pointing in the internal direction $\hat{1}$: $Q_R^a = |Q_R|\delta^{a1}$ and $Q_L^a = |Q_L|\delta^{a1}$.

At this point we can divide the fields into the ones that are excited in the black hole background (the dilaton, the metric, some moduli and some gauge fields) and the ones

that are not (the antisymmetric tensor, and the other moduli and gauge fields). Let the quadratic action including both types of fields be denoted

$$S_2 = S_{exc} + S_{rest}$$

We'll concentrate on $S_{rest}$ since it is much the simpler of the two. Its explicit form is

$$\begin{aligned} S_{rest} = & \frac{1}{32\pi} \int d^4x \sqrt{-g} [-\frac{1}{12} e^{-2\Phi} H_{\mu\nu\rho} H^{\mu\nu\rho} - \\ & - e^{-\Phi} \sum_{a,b \neq 1, d+1} F^{(a)}_{\mu\nu} (L\mathcal{M}L)_{ab} F^{(b)\mu\nu} + \frac{1}{8} Tr'(\partial_\mu \mathcal{M} L \partial^\mu \mathcal{M} L)] \end{aligned} \tag{3.20}$$

where the metric and dilaton are those of the black hole background. $Tr'$ is the usual trace, excluding the elements of the moduli matrix $\mathcal{M}_{ab}$ with $a, b \in (1, d+1)$. The reason for this index structure is not important for our discussion but it can be found easily from the form of the black hole solutions in [1,19].

We can now concentrate on the propagation of the antisymmetric tensor $B_{\mu\nu}$ or initially unexcited gauge fields $A_\mu$ or scalars $\mathcal{M}$, but for simplicity let us analyze the propagation of some element $\zeta$ of the scalar field matrix $\mathcal{M}$. The action will be

$$S = \frac{1}{256\pi} \int d^dx \{ -e^{2\Phi_d} \partial_0 \zeta \partial_0 \zeta + \partial_i \zeta \partial_i \zeta \} \tag{3.21}$$

and so the field equation is

$$e^{2\Phi_d} \partial_0^2 \zeta - \partial_i^2 \zeta = 0$$

Assuming a time dependence $\zeta(t, \vec{x}) = e^{-i\omega t} \zeta(\vec{x})$, and plugging in the black hole dilaton field dependence of (2.19), we get a Schrödinger equation with potential

$$V = \omega^2 (1 - e^{-2\Phi_d}) = -\frac{2Mc_d\omega^2}{|x|^{d-3}} - \left( \frac{\omega_{d-2}}{8\sqrt{2}\pi} \right)^2 \frac{(Q_R^2 - Q_L^2)\omega^2 c_d^2}{|x|^{2(d-3)}} \ , \tag{3.22}$$

leading to the following small angle scattering cross section:

$$\frac{d\sigma}{d\Omega} \sim \frac{M^2}{\theta^4} \ . \tag{3.23}$$

The simple form of the potential (3.22) can also be used to illuminate a previously known curiousity involving $d$ dimensional black holes. It is a familiar fact about the Schrödinger equation that sufficiently singular attractive potentials can lead to violations of unitarity in the form of partial or complete absorption of incoming waves. In the black

hole context, however, "absorption" just corresponds to falling through the event horizon and is perfectly acceptable. The potential (3.22) has different consequences in this respect in dimensions $d = 4$ and $d > 4$. For $d > 4$, the potential is so singular that there is a finite absorption probability for any energy. For $d = 4$, the potential is marginally singular and there is absorption only if the potential is stronger than a critical value. To be precise, a particle is completely reflected if its energy $\omega$ is less than a "mass-gap" defined by

$$(Q_R^2 - Q_L^2)\omega^2 < 1/8 \ .$$

This phenomenon had already been noted in the particular example of the four dimensional $a = 1$ dilaton black hole, in which case $Q_L = 0$ and $Q_R$ is essentially the black hole mass [24]. † Note that in the case $|Q_L| \geq |Q_R|$ the quadratic term in (3.22) is absent or repulsive and no modes, regardless of their energy, are absorbed. This is again consistent with the result in [24] that the mass gap is infinite for four dimensional dilatonic black holes with $a > 1$ ( $a = \sqrt{3}$ in this case)‡. This mass gap in four dimensions was used to reconcile the fact that the Hawking temperature of the extremal black hole is nonzero (!) with the picture of these black holes as elementary particles. In $d > 4$ the Hawking temperature vanishes [19] and this is consistent with the lack of a mass gap, as is implied by (3.22).

The analysis for the gauge fields and antisymmetric tensor field in (3.20), once the gauge is fixed, gives a similar action to (3.21) and the same small angle cross section (3.23) when we consider equal initial and final polarizations. The small angle scattering analysis of the fields in $S_{exc}$ also gives the same small angle cross section (3.23). This universal cross section, proportional to the mass of the black hole, corresponds to gravitational scattering.

## 4. String Tree Amplitude Calculations

We now turn to the calculation of extended string scattering amplitudes by string tree amplitude methods. Our goal is to establish a correspondence between particular states of the heterotic string and classical string solutions. Evidence for such a correspondence has

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† A curious fact is that when one inspects the supersymmetric part of the heterotic string tree amplitude for our kinematic situation, one sees that for $Q_L = 0$ the above threshold corresponds precisely to the energy necessary to excite the next mass level of the free string. This fact seems to be peculiar to four dimensions, however.

‡ We thank Finn Larsen for discussions on this issue

previously been found for ground state strings, but we will be able to extend it to a much larger class of states and in addition address the issue of polarization dependence. The lowest order scattering amplitudes were compared for a restricted class of four dimensional black holes and string states in [10].

The context is the heterotic string with the $X^9$ coordinate compactified on a macroscopic circle of radius $R_9$. We study strings which wind once around $X^9$, are in the right-moving ground state ($N_R = 1/2$), are in a general state of the left-moving oscillators ($N_L \geq 0$) and carry charge $q^I$ (and current). Such states are annihilated by half the supersymmetries and saturate the Bogomolny bound, since from the right moving point of view they correspond to massless ten dimensional states. Depending on whether we are in the NS or R sector we have bosons or fermions. Bosons of this type are created by vertex operators like

$$V = \xi.\psi_R \; p_R.\psi_R(z)\mathcal{O}_L(\bar{z})e^{i[\vec{p}_R\vec{X}(z)+\vec{p}_L\vec{X}(\bar{z})]} \tag{4.1}$$

where $\mathcal{O}$ is a left-moving operator of weight $N_L$ containing all the left-moving polarizations. The momenta are

$$p_L^\mu = (\hat{p}^{\hat{\mu}}, \frac{n}{R_9} - \frac{R_9}{2}, q^I) \qquad p_R^\mu = (\hat{p}^{\hat{\mu}}, \frac{n}{R_9} + \frac{R_9}{2})$$

where $\hat{p}$ is the nine-dimensional momentum and the next entry gives the momenta in the $X^9$ direction. The on-shell conditions on the ten-dimensional momenta, $0 = p_R^2 = p_L^2 + 2(N_L - 1)$, do two things: they fix the nine-dimensional mass-squared through the relation $\hat{p}^2 = (R_9/2 + n/R_9)^2$ and they constrain the total number of left moving oscillators (both coordinate and gauge) through $2(N_L - 1) = 2n - q^I q^I$. A given $N_L$ can usually be achieved by exciting many different combinations of oscillators and higher mass levels are increasingly degenerate.

We will study the two-body small-angle scattering of such particles using the method of [25] to express the tree-level heterotic string amplitude as the product of an open superstring amplitude for the right-movers and an open bosonic string amplitude for the left-movers:

$$\mathcal{A}^{het} \sim \sin(\pi u/2)\mathcal{A}_{ss}^{open}(s,u)\mathcal{A}_{bos}^{open}(u,t) \; . \tag{4.2}$$

These amplitudes are conveniently written in terms of the ten-dimensional right-moving Mandelstam variables:

$$s = -(p_R^1 + p_R^2)^2 \qquad t = -(p_R^1 + p_R^4)^2 \qquad u = -(p_R^1 + p_R^3)^2 \; .$$

We will use the convention that $\sum_i p_i^{R,L} = 0$. The superstring amplitude is simple:

$$\mathcal{A}_{ss}^{open}(s,u) = \frac{\Gamma(-s/2)\Gamma(-u/2)}{\Gamma(t/2+1)} K_{ss} \tag{4.3}$$

where $K_{ss}$ is a kinematic factor depending on the right-moving polarizations and momenta. The terms of direct relevance to us are

$$K_{ss} = -\frac{st}{4}(\xi_1.\xi_3)(\xi_2.\xi_4) + \left[\frac{1}{2}s(\xi_1.p_4)(\xi_3.p_2) + \frac{1}{2}t(\xi_3.p_4)(\xi_1.p_2)\right](\xi_2.\xi_4) + \ldots \tag{4.4}$$

(all momenta and polarizations here should carry a $R$ label). The bosonic factor is more complicated because of the left-moving oscillator vertices $\mathcal{O}_L$, but its general form can be worked out following [25]:

$$\begin{aligned}\mathcal{A}_{bos}^{open}(u,t) &= \sum_{\{n\}} C_{\{n\}} \int_1^\infty dx x^{p_R^1 \cdot p_R^2}(x-1)^{p_R^1 \cdot p_R^3} x^{-\tilde{n}_{12}}(x-1)^{-\tilde{n}_{13}} \\ &= \sum_{\{n\}} C_{\{n\}} \frac{\Gamma(-t/2-1+\tilde{n}_{12}+\tilde{n}_{13})\Gamma(-u/2-\tilde{n}_{13}+1)}{\Gamma(s/2+\tilde{n}_{12})}\end{aligned} \tag{4.5}$$

where the $\tilde{n}_{ij} = n_{ij} + p_i^R p_j^R - p_i^L p_j^L$ are certain integers: the $p_i^R p_j^R - p_i^L p_j^L$ part arises because, while this Veneziano integral is really built out of $p_L$, we chose to write it in terms of $p_R$; the $n_{ij}$ come from powers of $z_i - z_j$ arising from Wick-contracting the $\mathcal{O}_L(z_i)$ with each other and with the exponentials; the $C_{\{n\}}$ contain the left-moving polarizations hidden in the $\mathcal{O}_L$. Mutual locality of vertex operators in string theory guarantees that the $\tilde{n}$ are indeed integers.

We will study low velocity, small angle (large impact parameter) scattering of two of these string states. To keep the formulas simple, we take both nine-dimensional masses equal, but we let the left-moving momenta be different. In the center of mass, the ten-dimensional momenta can be written

$$\begin{aligned} p_R^1 &= (M\gamma, M\gamma\vec{v}, M) \qquad & p_R^2 &= (M\gamma, -M\gamma\vec{v}, M) \\ p_R^3 &= -(M\gamma, M\gamma\vec{w}, M) \qquad & p_R^4 &= -(M\gamma, -M\gamma\vec{w}, M)\end{aligned}$$

where $\gamma$ is the usual relativistic factor, $v^2 = w^2$ and $\vec{w}$ forms a small angle $\theta_c$. We denote by $\theta_c$ the scattering angle in the center of mass frame and $\theta = \theta_c/2$ is the scattering angle in the rest frame of string one, as defined in fig 1. with $\vec{v}$. The mass $M$ is related to

$N_L$ and $R_9$ as previously described. For small velocity and nearly forward scattering the Mandelstam invariants are:

$$s \sim -t \sim 4M^2v^2 \qquad\qquad u \sim -M^2v^2{\theta_c}^2 \ll 1$$

Note that we are not yet assuming that $s, t \ll 1$. We are interested in the singularities of (4.5) in the limit $u \to 0$ (large impact parameter).

The structure of (4.5) is such that such a $u$-channel pole can only appear if $\tilde{n}_{13} = 2, 1$. We need to find what $C_n$ is for these cases. The values of $\tilde{n}_{13}$ are set by the powers of $(z_1 - z_3)$ appearing the OPE of the left-moving vertex operators for particle 1 (one of the two incoming particles) and particle 3 (the particle into which it scatters). The first two terms in this OPE are, somewhat schematically,

$$\mathcal{O}_1(z_1)e^{ip_L^1 X(z_1)}\mathcal{O}_3(z_3)e^{ip_L^3 X(z_3)} \sim \frac{\delta_{13}\ e^{i(p_L^1+p_L^3)X(z_3)}}{(z_1-z_3)^{2-u/2}} + \frac{\mathcal{P}(z_3)\ e^{i(p_L^1+p_L^3)X(z_3)}}{(z_1-z_3)^{1-u/2}}$$

The first term generates contributions to the string tree amplitude with $\tilde{n}_{13} = 2$, while the second generates $\tilde{n}_{13} = 1$. There are some constraints which restrict the values of other $\tilde{n}_{ij}$'s. Since the conformal weight of each operator is one we have $\sum \tilde{n}_{ij} = 4$. When we send the point $z_4 \to \infty$ to fix the Möbius invariance of the amplitude and get (4.5), we pick the term with $\tilde{n}_{14} + \tilde{n}_{24} + \tilde{n}_{34} = 2$. This implies that $\tilde{n}_{12} + \tilde{n}_{23} = 2 - \tilde{n}_{13}$. Using the fact that $(p_1 + p_3)^{L,R}$ is of order $\theta_c$ we conclude that $n_{12} + n_{23} = 2 - \tilde{n}_{13}$. Since $n_{ij} \geq 0$ we conclude that $n_{12} = n_{23} = 0$ if $\tilde{n}_{13} = 2$, and one of them is 1 and the other zero if $n_{13} = 1$.

The leading term comes from the identity operator term in the OPE of $\mathcal{O}_1$ with $\mathcal{O}_3$. If we pick the polarization states $\mathcal{O}_i$ from an appropriately orthonormalized set, the answer is either 1 (polarizations of 1 and 3 the same) or 0 (polarizations of 1 and 3 orthogonal), up to terms of order $\theta_c$ due to the fact that $p_3^L$ is not precisely $-p_1^L$. This term is multiplied by the OPE of $\mathcal{O}_2$ with $\mathcal{O}_4$, which yields a Kronecker delta function of the polarizations of particles 2 and 4. The polarization operator $\mathcal{P}$ appearing in the next term in the OPE will have conformal weight one (one more than the identity operator in the leading term) and is thus of the form $\mathcal{P} \sim \partial X_L$ where $X_L$ is one of the 26 leftmoving bosons. This $\partial X_L$ should be contracted with the exponentials of the other two vertex operators, since conformal invariance implies that this three point function has poles of order one when $z_3 \to z_2, z_4$. This contraction will therefore be proportional to some component of $p_2^L + p_4^L$ which is of order $\theta$ and which reduces the term's order in $1/\theta$. In short, the most divergent term in the $\theta_c \to 0$ limit of $\mathcal{A}_{bos}^{open} \sim 1/{\theta_c}^2$ arises when the polarizations of the

scatterers are unchanged. The normalization of this term is known and is independent of the polarizations. The same thing is true of the $K_{ss}$ factor in $\mathcal{A}_{ss}^{open}$.

We are now ready to put together the net result for the leading $\theta_c \to 0$ behavior of the scattering amplitude with no polarization flip. We saw above that $n_{12} = 0$ so that $\tilde{n}_{12} = p_1^R p_2^R - p_1^L p_2^L \equiv \Delta_{12}$, which reduces to $N_L - 1$ if we are scattering two identical strings.

With these facts in hand, we can see that

$$\begin{aligned}
\mathcal{A}_{ss}^{open} \sim& \frac{s}{u} \\
\mathcal{A}_{bos}^{open} \sim& \frac{1}{u}\frac{\Gamma(-t/2+\Delta_{12}+1)}{\Gamma(s/2+\Delta_{12})} \sim \frac{1}{u}(\Delta_{12}+s/2) \\
\mathcal{A}^{het} \sim& u\, \mathcal{A}_{ss}^{open}\mathcal{A}_{bos}^{open} \sim \frac{\Delta_{12}}{{\theta_c}^2}
\end{aligned}$$

The expression for $\mathcal{A}^{het}$ is a typical "metric on moduli space" result; the angular distribution has a well-defined limit as $v \to 0$. In this limit, for $\Delta_{12} \neq 0$, this implies an interaction Hamiltonian proportional to $v^2\Delta_{12}$ in agreement with (3.7). Somewhat remarkably, it reproduces the classical string solution scattering result (3.9) for *any* oscillator level (in [9], this result was shown for the lowest level only). For two identical strings and $N_L - 1 = 0$, the leading term vanishes ($\Delta_{12} = 0$ and the metric on moduli space is flat) and we find

$$\mathcal{A}^{het} \sim \frac{M^2 v^2}{{\theta_c}^2}$$

in agreement, once again, with the classical result (3.11). We can also analyze the case of massless particle scattering by a black hole. The kinematic configuration is

$$\begin{aligned}
p_R^1 &= (M, \vec{0}, M) \qquad & p_R^3 &= -(M\gamma, M\gamma\vec{v}, M) \\
p_2 &= (E, E\hat{x}, 0) \qquad & p_4 &= -(E', E'\hat{n}, 0)
\end{aligned}$$

and in the limit $E << M$ we have $E \sim E'$ and

$$s \sim -t \sim 2ME \qquad u = -E^2{\theta_c}^2$$

for small $\theta_c$. An analysis similar to the above tells us that the leading small-angle amplitude is diagonal in polarization and independent of the specific polarization value. The amplitude is

$$\mathcal{A}^{het} \sim \frac{M^2}{{\theta_c}^2}$$

in agreement with (3.23).

Now we turn to the polarization flipping amplitudes. We saw above that all polarization flipping amplitudes are at least one factor $\theta_c$ down with respect to the polarization preserving ones. We will analyse the leading case in which the amplitude goes as $\mathcal{A} \sim 1/\theta_c$. In this limit either the right or the left moving polarization changes but not both.

Let us start with right moving polarization changes. We see from (4.4) that the polarizations have to be in the scattering plane. Let us assume, as we did in section 4.2, that the polarizations are $\xi_1^R = \hat{3}, \xi_3^R = \hat{2}$ and $\xi_2^R = \xi_4^R$. All right moving polarization dependence of the amplitude is in the factor (4.4) so that

$$\frac{\mathcal{A}^{flip}}{\mathcal{A}^{no\ flip}} = \frac{K_{ss}^{flip}}{K_{ss}^{noflip}} = 2\theta$$

independent on the value of $N_L$ of both strings, in agreement with the semiclassical result (3.13).We can also calculate the polarization flipping amplitudes for the fermionic states, which are characterized by a spinor $u_a^R$. We can verify that also for this case there is an agreement with the semiclassical calculation.

Now we turn to the left moving oscillators, starting with the $N_L = 1$ case. For this case the full amplitude involves a polarization dependent factor $K_{bos}$ which, for the terms we are interested in, reduces to (4.4) but in terms of the left moving momenta and Mandelstam variables. Taking again $\xi_1^L = \hat{3}, \xi_3^L = \hat{2}$ with $\xi_2^L = \xi_4^L$ and $p_1^L \neq p_2^L$, we have that $\Delta_{12}$ is nonzero. We find

$$\frac{\mathcal{A}^{flip}}{\mathcal{A}^{no\ flip}} = \frac{K_{bos}^{flip}}{K_{bos}^{noflip}} = \frac{v^2 M^2 \theta}{\Delta_{12}}$$

in agreement with (3.19). If the two strings are identical we find that this ratio is $2\theta$, as we found also in the classical calculation.

We have also found agreement for the case $N_L = 2$. The calculation for higher $N_L$ seems more cumbersome.

## 5. Discussion and Conclusions

In this work we have studied in some detail multiple classical oscillating fundamental string solutions of ten dimensional heterotic string theory which possess mass, $U(1)$ charges, and longitudinal momentum per unit length. The oscillations are left moving and the solutions possess unbroken target space supersymmetry. Upon toroidal compactification

along with an appropriate averaging procedure, these solutions yield the full complement of static extremal electrically charged black holes in lower dimensions previously found in the literature. The new solutions which we obtained may include, in addition, configurations that could have interesting physical interpretations other than the particular ones explored here. One issue that we leave for future work is whether there is a correspondence between $d$ dimensional BPS-saturated rotating black holes and string states similar to what we have found for the static case. A step in this direction has been taken in [26].

In our investigations of the classical solutions we found that the requirement that curvature singularities be invisible to outside observers imposed one constraint on the parameters of the classical solutions. This invisibility requirement is necessary for any classical solution to be physically reasonable. Remarkably, this condition was found to be identical, up to a classically undetectable normal ordering constant, to the level matching condition for the fundamental string corresponding to the classical solution. The oscillations are characterized by eight transverse arbitrary functions, corresponding to the eight transverse physical polarization degrees of freedom of the string states.

It was argued that these multiple oscillating fundamental string solutions were exact to all orders in the string tension in some scheme and, since the string coupling does not blow up anywhere for these electric solutions, loop corrections are expected to be small. The higher dimensional origin of the $d$ dimensional black holes then implies that the same things can be said of the classical black hole solutions.

The relationship between these classical oscillating strings and fundamental strings was explored via comparison of two-body scattering amplitudes, in the semiclassical approximation for the oscillating string solutions and at tree level in string theory. It was found that the low velocity small angle scattering cross sections agreed, and in particular that the lowest order left and right handed polarization flipping amplitudes agreed in the two different approaches. This we regard as additional evidence that the classical solutions should be regarded as the fundamental strings themselves. It should be noted that this evidence was dynamical and not simply a consequence of kinematics.

By using the above direct connection between compactified strings and black holes, we computed two body black hole scattering in the lower dimensional theory and found that the result again agrees with that obtained from string theory. In an investigation of scattering of massless particles off a black hole, agreement was also found in the two different approaches. The leading order dynamics of the black holes was found to be independent of their internal states.

I has been shown [3,19] that the entropy of the $d$ dimensional extremal black holes, calculated at the "stretched horizon" according to the usual area rule, scales in the same way as the entropy of right-moving ground state strings with increasing total left moving oscillator level. In these investigations it was unclear precisely which black hole degrees of freedom the entropy was counting. Here, by contrast, we saw that extremal electrically charged black holes in $d = 4 \ldots 9$ should be thought of as compactified strings. Therefore in order to discover the real internal structure of the $d$ dimensional black hole, which is a solution of the low energy effective action of string theory, one should do measurements of the fields with resolution better than the compactification radius. At this point, the lower dimensional effective action description breaks down, and the ten dimensional nature of the theory becomes manifest. The correspondence between the ten dimensional string and the lower dimensional black hole then makes those degrees of freedom which account for the entropy manifest. In this regard, the important length scale is the compactification scale and not, for example, $\sqrt{\alpha'}$. In other words, in order to differentiate between two black holes with the same charges we should measure the fields with a resolution better than the compactification scale. On the other hand, if we insist on a $d$ dimensional description, these oscillations along the internal directions of the ten dimensional fields are viewed as massive fields in the $d$ dimensional theory. For a given value of the charges, different black holes correspond to different ways of exciting these massive fields.

All of the string states and classical objects studied were supersymmetric, and under these conditions it is expected that there are nonrenormalization theorems available to protect the relation between the mass and certain charges. In order for the physical properties of four dimensional black holes to be studied, however, one needs to know what happens outside the protected enclosure formed by supersymmetry. An example would be the excitation spectrum of the black hole, which we expect to be quite different from the free string spectrum, because of large gravitational corrections.

Despite the fact that quantum corrections to the classical backgrounds are formally under control, the physics of the classical string singularity remains opaque to us. Possibly the investigation [27] may shed light on this issue. In general, the external fields of a configuration do not necessarily tell us much about its real internal structure, but here we have found some evidence that a classical oscillating string should be thought of as the fundamental string itself, and thus that BPS-saturated black holes should be thought of as compactified fundamental strings. In this context it would be interesting to explore the issue of pair production of black holes. We leave for the future the question of how

much more information may be obtained regarding black holes, for example non-extremal ones, by using string theory. It is our hope that the black hole information problem will be resolved in the context of string theory.

In the context of string-string duality there is an interesting application of the solutions we have found. We have in mind the duality between the heterotic string on $T^4$ and type IIA on K3. It is well known that starting from the fundamental string solution of the form (2.1) in six dimensions ($\Lambda \sim 1/r^2$) and applying the duality transformation we obtain a non singular soliton solution of type IIA on K3 [28,29] . Applying this same transformation to our oscillating string solution we can construct oscillating nonsingular soliton string solutions of type IIA on K3. The condition that the solution be nonsingular in the type IIA theory is in fact the same as the condition that the singularity be non naked in the heterotic theory and is the the level matching condition for heterotic string states. This establishes the correspondence of type IIA solitons with the heterotic string beyond the small oscillation zero mode analysis of [29]. We intend to explore this possibility further.


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## 7. Appendix: Conformal Invariance

Here we investigate the model

$$\begin{aligned} S =& \frac{1}{\pi\alpha'} \int d^2z \left\{ F(x,u)\partial u[\bar{\partial}v + K(x,u)\partial u + 2V_i(x,u)\bar{\partial}x^i] \right. \\ & \left. +\partial x_i \partial x^i + \frac{\alpha'}{8}\mathcal{R}^{(2)}\Phi(x,u) \right\} \end{aligned} \tag{7.1}$$

By doing a coordinate transformation we can set $K = 0$, from now on we set it to zero. We will follow closely the method of ref. [6], generalizing it to the case when $F$ depends also on $u$. We want to show that

$$Z(V,U,X) = \int \mathcal{D}(u,v,x^i) e^{-S - \frac{1}{\pi\alpha'}\int d^2z V(z)\partial\bar{\partial}u + U(z)\partial\bar{\partial}v + X^i(z)\partial\bar{\partial}x^i}$$

is conformally invariant. We have included some source terms for the fields. We want to find the conditions for which this partition function is independent of the conformal factor of the two dimensional metric $g = e^{2\varphi}\eta$.

We start doing the integration over $v$, which produces a factor

$$\delta\left(\bar{\partial}[F\partial u - \partial U]\right) = \frac{\delta(u - u^{cl})}{Det[F(x,u^{cl})]Det[\bar{\partial}\partial - \bar{\partial}\partial_u F^{-1}(x,u^{cl})\partial U]} \tag{7.2}$$

where $u^{cl}$ is the solution to the equation

$$F(x,u^{cl})\partial u^{cl} = \partial U \tag{7.3}$$

This equation can be solved iteratively, assuming $U$ small, as follows

$$u = u^0 + \sum_{n=1}^{\infty}(u^n - u^{n-1})$$

$$u^n = \frac{1}{\partial}F(x,u^{n-1})^{-1}\partial U$$

where $u^0$ is a constant and $u^n$ contains powers of $U$ up to $U^n$.

$$u^{cl} = u^0 + \frac{1}{\partial}F(x,u^0)^{-1}\partial U + \frac{1}{\partial}\partial_u F(x,u^0)^{-1}\partial U\frac{1}{\partial}F(x,u^0)^{-1}\partial U + \cdots$$

If we think of these functions in terms of Feynman diagrams, by replacing the inverse powers of $\partial$ by lines, we notice that the vertices are of the form $\partial_u^n F^{-1}\partial U$. The determinant of the function $F$ has been calculated in [30][31] [7]

$$\frac{1}{Det[F]} = const \times \exp\left[-\frac{1}{4\pi}\int d^2 z R^{(2)}(-\frac{1}{2}\log F) + \frac{1}{2\pi}\int d^2 z\bar{\partial}\log F\partial\log F\right] \tag{7.4}$$

which has the effect of redefining the dilaton in (7.1)to $\Phi' = \Phi - \log F$. The second term in the exponential (7.4) can be eliminated by adding to the original action (7.1) an order $\alpha'$ counterterm which effectively amounts to changing the metric to $G_{\mu\nu} \to G_{\mu\nu} + \frac{\alpha'}{2}\partial_\mu \log F\partial_\nu \log F$. We can view this as a field redefinition (note that $F^2 = \det G$) or a choice of scheme [32]. The second determinant in (7.2) can be written introducing a pair of auxiliary bosonic fields $y^a,\ a = 1,2$ as

$$\begin{aligned} &Det[\bar{\partial}\partial - \bar{\partial}\partial_u F^{-1}(x,u^{cl})\partial U]^{-1} = \\ =&(const.)\int \mathcal{D}(y)e^{-\frac{1}{\pi\alpha'}\int\{\bar{\partial}y^a\partial y^a - \bar{\partial}y^a\partial_u F^{-1}(x,u^{cl})\partial U y^a\}} \end{aligned} \tag{7.5}$$

The integration over $u$ can be readily done. We obtain the effective action

$$S_{eff} = \frac{1}{\pi\alpha'} \int d^2z \, \{ \partial U 2V_i(x,u) \bar{\partial} x^i - \bar{\partial} y^a \partial_u F^{-1}(x, u^{cl}) \partial U y^a + \\ + \partial x_i \bar{\partial} x^i + \partial y^a \bar{\partial} y^a + \frac{\alpha'}{8} \mathcal{R}^{(2)} \Phi'(x,u) - \bar{\partial} V F^{-1}(x,u) \partial U + X^i \partial \bar{\partial} x^i \Big\} \tag{7.6}$$

where we understand that, from now on, where it says $u$ we mean $u^{cl}$. Note that we have used (7.3) to express some of the terms in (7.6). We have to integrate (7.6) over $x^i$ and $y^a$. The effective action (7.6) has, already at the classical level, a dependence on the conformal factor of the metric, coming from the dilaton term through $R^{(2)} = -2\bar{\partial}\partial\varphi$. Note that the action (7.6) is non local because of the dependence of $u$ on the other fields of the theory. We can however expand the action in terms of Feyman diagrams and analyze the divergences. This action (7.6) has the interesting property that all vertices contain a derivative $\partial$ acting on a background classical field (e.g. $\partial U$ ) so that in a quantum loop involving $n$ vertices we get an integral which, in the worst case goes as $\int d^2p \bar{p}^{2n}/p^{2n+2}$ which is not divergent because of its tensor structure (Pauli-Villars regularization, for example, respects this tensor structure). We conclude that the only divergent terms are the ones coming from tadpole diagrams. When we regularize these divergences we find an extra dependence on the conformal factor of the metric. We will use the heat kernel definitions for contractions at coinciding points [33]

$$\langle x(z)x(z)\rangle = \frac{\alpha'}{2}\varphi(z) \qquad\qquad \langle\bar{\partial}x(z)x(z)\rangle = \frac{\alpha'}{4}\bar{\partial}\varphi(z) \tag{7.7}$$

We analyze the $\varphi$ dependence of vertices including different combinations of background fields. First we will consider the term $\bar{\partial} V F^{-1} \partial U$. Tadpoles will vanish if

$$\partial_i \partial_i F^{-1} = 0 \tag{7.8}$$

This equation ensures also that tadpole contractions in all vertices defining $u$ also vanish since they involve always $F^{-1}$ or derivatives of it. Let us now analyze the classical dilaton contribution

$$\frac{1}{8\pi}\int(-4\varphi)\bar{\partial}\partial\Phi' = \frac{1}{8\pi}\int(-4\varphi)\, \{ \partial_u \partial_i \Phi' (\bar{\partial} x^i F^{-1} \partial U + \partial x^i \bar{\partial} u) + \\ \partial_i \Phi' \bar{\partial}\partial x^i + \partial_u^2 \Phi' F^{-1} \partial U \bar{\partial} u + \partial_u \Phi' \partial_u F^{-1} \partial U \bar{\partial} u + \\ \partial_u \Phi' \partial_i F^{-1} \bar{\partial} x^i \partial U + \partial_u \Phi' F^{-1} \partial\bar{\partial} U + \partial_i \partial_j \Phi' \bar{\partial} x^i \partial x^j \} \tag{7.9}$$

which implies

$$\partial_i \partial_j \Phi' = 0 \qquad\qquad \partial_u \partial_i \Phi' = 0 \tag{7.10}$$

since the terms $\partial x^j \bar{\partial} x^i$ or $\partial x^i \bar{\partial} u$ cannot be produced from quantum corrections. This implies that $\Phi' = -z(u) + b_i x^i$. For simplicity we will set to zero the linear terms. Terms proportional to $\partial \bar{\partial} U$ vanish on shell.

Tadpole contractions of the vertex $\partial U 2V_i \bar{\partial} x^i$, and the use of (7.7), give

$$\begin{aligned} &\frac{1}{\pi\alpha'} \int d^2 z \partial U 2V_i(x,u) \partial x^i |_{tadpoles} = \\ &= \frac{1}{2\pi} \int d^2 z \varphi \left\{ (\partial_j^2 V_i - \partial_i \partial_j V_j) \partial U \bar{\partial} x^i - \partial_i V_i \partial \bar{\partial} U - \partial_u \partial_i V_i \partial U \bar{\partial} u \right\} \end{aligned} \tag{7.11}$$

There are also terms coming from the vertex in (7.5)

$$\begin{aligned} &-\frac{1}{\pi\alpha'} \int d^2 \bar{\partial} y^a \partial_u F^{-1}(x, u^{cl}) \partial U y^a |_{tadpoles} = \\ &\frac{1}{2\pi} \int d^2 z \varphi \left\{ \partial_u^2 F^{-1} \partial U \bar{\partial} u + \partial_i \partial_u F^{-1} \partial U \bar{\partial} x^i + \partial_u F^{-1} \partial \bar{\partial} U \right\} \end{aligned} \tag{7.12}$$

Collecting all these terms together and demanding that the coefficients of $\partial U \bar{\partial} u$ and $\partial U \bar{\partial} x^i$ vanish we obtain the equations

$$\begin{aligned} &\partial_u \partial_i V_i - \partial_u^2 F^{-1} + \partial_u F^{-1} \partial_u \Phi' + F^{-1} \partial_u^2 \Phi' = 0 \\ &\partial_j \partial_j V_i - \partial_i \partial_j V_j + \partial_i \partial_u F^{-1} - \partial_i F^{-1} \partial_u \Phi' = 0 \end{aligned} \tag{7.13}$$

These are indeed the equations that we obtained in (2.12). One can also have a solution with a linear dilaton $\Phi' = -z(u) + b_i x^i$, in which case the equations (7.13) and (7.8) acquire some extra terms, which come from the fact that with a linear dilaton, the one point function $\langle x \rangle$ is non zero.

This completes the argument showing conformal invariance of this generalized sigma model. Summarizing, the background has to satisfy (7.8) (7.10) (7.13).

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